Application of genetic algorithms to the development of a variable Schmidt number model for jet-in-crossflows

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Abstract *Proposes the use of genetic algorithms to assist the development of turbulence models. A variable Schmidt number model for scalar mixing in jet-in-crossflows was developed through theoretical analyses. A uniform micro genetic algorithm is implemented to optimize the model. This is the first known application of the genetic algorithm (GA) technique to turbulence model development. Overall, the GA technique worked exceptionally well for this problem in a cost-effective and time-efficient manner. A set of experimental data on a single round jet issued into a confined crossflow is selected for calibration and optimization of the model constants using the uniform micro-genetic optimization algorithm. Three sets of experimental data of jet-in-crossflows are used for the validation of the new model. Numerical results show that the proposed scheme of using the genetic algorithms to develop turbulence models produces very promising results.*

Introduction

Genetic algorithms (GA) have been used extensively in design optimizations. In the present article, we propose the use of GA to assist the development of turbulence models. As is well known, there is no one existing turbulence model that can be used to simulate all turbulent flow problems. Engineering design often have to rely on special models that are tailored for the specific applications. In the present work, we describe a procedure by which we developed a turbulence mixing model for jet-in-crossflows with the help of GA.

Jet-in-crossflows is used extensively in gas turbine combustors to enhance combustion efficiency and reduce combustor exit temperature. The accurate prediction of scalar mixing in a jet-in-crossflow is critical to the design process, especially so for low emission combustor design. The current practice in the aircraft engine industry for the numerical simulations of turbulent combustion relies on the k-ε model with the turbulent diffusion of scalars modeled through the use of a constant Schmidt number. Conventionally a Schmidt number around 0.7 is being used. However, our previous study (He *et al.*, 1999) shows that this assumption is inadequate for combustor simulations, and a major revision on the concept of constant Schmidt number is needed.

The concept of a variable Schmidt number has been previously addressed in various contexts by other researchers (Reynolds, 1975). However, the earlier works were based mostly on mixing length models, and much attention was given to the near wall behavior of the turbulent Schmidt number. Work on complex

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flows, such as jet-in-crossflow, based on more up-to-date turbulence models is lacking. Recent experimental studies (Smith and Mungal, 1998) show that, in addition to the counter-rotating vortex pair, the wake structure and the large scale, intermittent structures in jet-in-crossflows all play important roles in the mixing process. Compared to the conventional constant Schmidt number model, a variable Schmidt number model that includes more flow physics is believed to have the capability to model the above-mentioned phenomena more adequately.

Based on theoretical analyses, a variable Schmidt number model that includes the effects of mean strain and turbulence properties has been developed in the present work. There are three model constants that need to be determined for this model. To search this large multimodal parameter space efficiently, a robust GA technique (Goldberg, 1989; Carroll, 1996) was implemented to find the set of unknown parameters which best matched predictions of scalar transport with experimental data. Experimental data of a single round jet issued into a confined crossflow by Kamotani and Greber (1972, 1974) is selected for model calibration and optimization. Three sets of experimental data of jet-in-crossflows by Kamotani and Greber (different momentum ratio), Crabb *et al.* (1981) and Sherif and Pletcher (1989a, b) are used for model validation.

Variable Schmidt number model

Reynolds-averaged scalar transport equation can be written as:

$$\frac{\partial Y_s}{\partial t} + U_j \frac{\partial Y_s}{\partial x_j} = D \frac{\partial^2 Y_s}{\partial x_j^2} + \frac{\partial}{\partial x_j}\left(-\overline{Y_s' u_j}\right) \tag{1}$$

in which, U_j and Y_s are mean velocity components and scalar concentration, respectively. D is the molecular diffusion of the scalar. The scalar flux $(-\overline{Y_s' u_j})$, which needs to be modeled, is a second order correlation of fluctuating scalar concentration and fluctuating velocity.

To model the scalar flux properly, we begin with its transport equation. Assuming large Reynolds number and neglecting the effect of buoyancy, the scalar-flux transportation equation can be written as:

$$\begin{aligned} \frac{\partial}{\partial t}\overline{Y_s' u_i} + U_j \frac{\partial}{\partial x_j}\overline{Y_s' u_i} &= -\frac{\partial}{\partial x_j}\left[\overline{Y_s' u_i u_j} + \frac{1}{\rho}\overline{p Y_s'}\delta_{ij}\right] \\ &\quad - \left[\overline{Y_s' u_j}\frac{\partial U_i}{\partial x_j} + \overline{u_i u_j}\frac{\partial Y_s}{\partial x_j}\right] + \overline{\frac{p}{\rho}\frac{\partial Y_s'}{\partial x_i}} - (\nu + D)\overline{\frac{\partial u_i}{\partial x_j}\frac{\partial Y_s'}{\partial x_j}} \end{aligned} \tag{2}$$

in which, terms on left hand represent the time change rate and convection of the scalar flux. The first term on right hand represents the flux transport caused by fluctuating velocity and fluctuating pressure. The second term on right hand represents the increase of transport flux caused by rate of mean shear strain and mean scalar concentration gradient. The third term is a correlation term of fluctuating pressure and fluctuating scalar concentration, and it causes a redistribution of flux components in three directions. The fourth term represents the flux dispersion caused by the molecular motion. The first, third, and fourth terms on the right-hand side need to be modeled.

For large Reynolds number, steady, homogeneous turbulent flows, the unsteady term can be neglected and Equation (2) can be simplified as:

$$\left[\overline{Y_s' u_j}\frac{\partial U_i}{\partial x_j} + \overline{u_i u_j}\frac{\partial Y_s}{\partial x_j}\right] = \frac{1}{\rho}\overline{p\frac{\partial Y_s'}{\partial x_i}}. \tag{3}$$

The pressure-scalar concentration gradient correlation term, $\frac{1}{\rho}\overline{p\frac{\partial Y_s'}{\partial x_{xi}}}$, can be split into two terms, return-to-isotropy term Π^s and rapid pressure-scalar concentration gradient term Π^r. Following common practice in standard k-ε modeling (Launder and Spalding, 1974), these two terms can be modeled as (Chen, 1991):

$$\Pi^s = -C_1\overline{Y_s' u_i}\frac{\varepsilon}{2k} \tag{4}$$

$$\Pi^r = C_2\overline{Y_s' u_j}\frac{\partial U_i}{\partial x_j} - C_3\overline{Y_s' u_j}\frac{\partial U_j}{\partial x_i} \tag{5}$$

Thus equation (3) can be rewritten as:

$$AS_{ij}\overline{Y_s' u_j} = -\overline{u_i u_j}\frac{\partial Y_s}{\partial x_j} \tag{6}$$

in which,

$$S_{ij} = \frac{\partial U_i}{\partial x_j} + B\frac{\partial U_j}{\partial x_i} + C\frac{\varepsilon}{2k}\delta_{ij}. \tag{7}$$

Assuming isotropic turbulence, equation (6) can be simplified as:

$$AS_{ij}\overline{Y_s' u_j} = -\frac{2k}{3}\frac{\partial Y_s}{\partial x_i}. \tag{8}$$

In order to avoid matrix inverse, let tensor S_{ij} be its determinant times a unit tensor. Thus equation (6) can be simplified as:

$$-\overline{Y_s' u_i} = \frac{4}{3AC}\frac{k^2}{\varepsilon}\frac{1}{\left|\tilde{S}_{ij}\right|}\frac{\partial Y_s}{\partial x_i} \tag{9}$$

in which,

$$\tilde{S}_{ij} = S_{ij}\frac{2k}{C\varepsilon}. \tag{10}$$

Schmidt number can be expressed as, $Sc = \frac{\nu_t}{D_t}$, in which ν_t and D_t denote turbulence eddy viscosity coefficient and turbulence diffusion coefficient, respectively. From standard k-ε model, ν_t can be written as:

$$\nu_t = C_\mu k^2/\varepsilon \tag{11}$$

where $C_\mu = 0.09$.

From equation (9), we get:

$$D_t = \frac{4}{3AC}\frac{k^2}{\varepsilon}\frac{1}{\left|\tilde{S}_{ij}\right|}. \tag{12}$$

Thus a variable Schmidt number model can be formulated as:

$$Sc = \frac{3AC}{4}C_\mu\left|\tilde{S}_{ij}\right|. \tag{13}$$

Three new constants, A, B and C are introduced and need to be determined. From equation (10), we can see that this model includes effects of the mean shear, effects of turbulent kinetic energy k and its dissipation ε on scalar transport in a jet-in-crossflow. Constants A and C are proportional to the magnitude of Schmidt number. Constant C also reflects the relative importance of turbulence frequency scale ε/k to the mean shear. Constant B reflects the effects of asymmetry, when B equals 1, both S_{ij} and $\tilde{S}_{ij}$ are symmetric tensors.

Application of genetic algorithms for model optimization

Preliminary calculations with the above variable Schmidt number model (using given sets of parameters, A, B and C, that were believed to be reasonable guesses) showed that the parameter space was multimodal, i.e. there are a very large number of local minimums in this parameter space. The question now becomes, how do we efficiently search this multimodal parameter space for a combination of these three parameters which provide the optimum overall agreement with experimental data?

The current literature identified (Goldberg, 1989) three main types of search methods: calculus-based, enumerative, and random. Calculus-based methods have been studied extensively, and have many successful applications. However the methods are local in scope, and depend upon the restrictive requirements of continuity and derivative existence. Enumerative schemes have been considered in many shapes and sizes. The idea is fairly straightforward; with a finite search space, or a discretized infinite search space, the search algorithm starts looking at objective function values at every point in the space, one at a time. Although the simplicity of this type of algorithm is attractive, and enumeration is a very human kind of search (when the number of possibilities is small), such schemes lack efficiency. Since the variable Schmidt number model is nonlinear and the large parameter space is multimodal, calculus-based and enumerative techniques were discounted for either robustness or efficiency. A GA technique, which is a search procedure that uses random choice as a tool to guide a highly exploitative search through a coding of a parameter space (Goldberg, 1989), is chosen to deal with this optimization problem.

A uniform micro-GA procedure, recommended by Carroll (1996) for its simplicity, robustness and efficiency, was implemented in the current CFD code (He *et al.*, 1999). A brief description of the GA technique will be presented, followed by a more detailed description of the technique applied to the current parameter optimization problem.

GAs are based on biological natural selection, evolutionary processes, and natural genetics. An initial population of size *n* is created from a random selection of the parameters in the parameter space. Each parameter set represents the individual's chromosomes. Each of the individuals is assigned a fitness based on how well it performs in its environment. Generally, there are three GA operators to create new generation:

(1) selection;

(2) crossover; and

(3) mutation.

Fit individuals are selected for mating, while weak individuals die off. Mated parents create a child with a chromosome set that is some mix of the parents' chromosomes. Then there is a small probability that one or more of the child's chromosomes will be mutated. The process of mating and child creation is continued until an entire new population of size *n* is generated, with the hope that strong parents will create a fitter generation of children. In practice, the average fitness of the population tends to increase with each new generation. Population convergence is defined to occur when the chromosome of all the individuals differ from that of the best fit individual in only a small fraction of the chromosome bits. When population convergence occurs while the best individual does not have the highest fitness of all possibilities, it is called a premature convergence. To avoid premature convergence, a micro-GA (Carroll, 1996) is adopted. A new random population is chosen while keeping the best fit individual from the previously converged generation and the evolution restarts. Successive generations are created until, to the extent allowed by practical constraints, a very fit individual is obtained. To assist the understanding of this GA procedure, a pseudo-code for the model parameter optimization problem is shown in Figure 1.

Tournament selection, no mutation, elitism and uniform crossover with micro-GA were used in the application. For completeness, a brief outline of each is given below:

- *Tournament selection*. Random pairs are selected from the population and the fitter of each pair is allowed to mate. The process of selecting random pairs and mating the stronger individuals continues until a new generation of size *n* is repopulated.
- *Mutation*. This is the occasional (with small probability) random alteration of a gene. It may reintroduce useful genetic material, which is lost through selection and crossover, to the chromosomes.
- *Élitism*. After a population is generated, the GA checks to see if the best parent has been replicated; if not, then a random individual is chosen and

```
Read control parameters for the GA procedure;
generation:=1;
Initial population with random binary strings;
while generation ≤ max_generation do
   Evaluate the fitness of all individuals;
   Find the best individual;
   for i=1 to population_size step 2 do
      Perform tournament selection;
      Perform uniform crossover;
   endfor
   Copy the offspring into new population;
   Reproduce the best parent into a random slot;
   Check the convergence of the new population;
    if premature then
      Reproduce the best individual;
      Regenerate other individuals randomly;
    endif
    generation:=generation+1;
endwhile
```

Figure 1. Pseudo-code of the genetic algorithm for the model parameters optimization problem

the chromosome of best parent is reproduced into that individual. This operator can help prevent the random loss of the best chromosome string.

- *Uniform crossover*. Every bit of two selected parent chromosomes will be determined randomly if they need to be exchanged between these two chromosomes. In uniform crossover, it is possible to obtain any combination of two parents' chromosomes. A crossover probability was chosen to be $Pc = 0.5$.

For the representation of the individuals (chromosomes), binary strings are chosen. A specified parameter interval $[U_{min}, U_{max}]$ is mapped into an n-bits binary string linearly (see Figure 2), in which U denotes parameters A, B or C. In this way, we can carefully control the range and precision of the unknown parameter. The error of this mapped coding can be calculated as:

$$\varepsilon = \frac{U_{\max} - U_{\min}}{2^n - 1} \tag{14}$$

where n denotes the number of bits used to represent the parameter.

In the current application, the length of each chromosome is 30 bits, with ten bits dedicated to each of the model parameters A, B, and C

0 0 0 0 ⟶ *Umin*
1 1 1 1 ⟶ *Umax*
others map linearly in between

Figure 2. Mapping of a single U parameter for $n = 4$

(see Figure 3). That means there are $1,024(2^{10})$ possibilities for each parameter, and $1,024^3$ possibilities for the parameter combinations. Based on preliminary calculations, the ranges of three model parameters are judiciously chosen as follows: A from 0.0 to 10.0, B from –5.0 to 5.0 and C from 0.0 to 5.0. From equation (14), we can see that the errors of this mapped coding will be 0.01, 0.01 and 0.005 for parameters A, B and C, respectively.

The selection of the fitness function is also important. It may significantly affect the history of convergence. In the current problem, the target is maximizing the agreement of our numerical prediction with experimental data. We assume the velocity field and pressure field, which in the present study is not significantly affected by the Schmidt number, have been simulated correctly, and error mainly comes from the new Schmidt number model. So only the agreement of the scalar concentration distribution is considered.

We have tested two fitness functions, which are defined as:

$$fitness_a = 1 - \left\{ \frac{1}{m} \sum_m (Y_{sc} - Y_{se})^2 \right\}^{1/2} \tag{15}$$

$$fitness_r = 1 - \left\{ \frac{1}{m} \sum_m \left(\frac{Y_{sc} - Y_{se}}{Y_{se} + \sigma} \right)^2 \right\}^{1/2} \tag{16}$$

where Y_s denotes the mean scalar concentration, subscripts c and e denote numerical calculation and experimental measurement, respectively, m is the number of numerical data adopted for comparison with experimental data, σ is a small positive constant, in this application $\sigma = 0.001$. Note that in both formulas, smaller total difference of scalar concentration represents better fitness. When the accuracy in the large scalar concentration region is of primary concern, equation (15), which reflects an absolute error, is more appropriate as a fitness function, while equation (16), which reflects a relative one, is more appropriate when the accuracy in the low scalar concentration region is more important.

For mixing in the jet-in-crossflow, it is relatively difficult to obtain a good numerical prediction in the region of low scalar concentration, where it is most important for combustion simulation. The fitness function given in equation (16), which has advantage of exaggerating the importance of prediction of low scalar concentration, is adopted in the present study for the calibration and optimization of the model parameters.

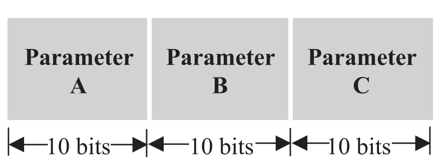


Figure 3.
Chromosome representation of the genetic algorithm for the model calibration problem

Numerical calibration and optimization

The numerical simulation is performed using the Reynolds-averaged Navier-Stokes equations coupled with a standard k-ε model (Sherif and Pletcher, 1989b) and the present variable Schmidt number model. A SIMPLE algorithm (Chen, 1991), with a second order hybrid finite volume scheme, was adopted. In order to stabilize the solution, under-relaxation factors were used for primitive variables.

Experimental data of a single heated round jet issued into a confined crossflow, by Kamotani and Greber (Kamotani and Greber, 1972, 1974), was selected for the model parameter calibration. The temperature difference between the jet and the crossflow is 167K, with the jet temperature at 465K. The distance from the jet exit to the opposite wall is H = 12D for momentum ratio J = 32, where D denotes the jet diameter. The velocity of the air crossflow is 8m/s.

Based on symmetry about the jet center plane, the computational domain was established on half of the flowfield. In this computational domain, a grid of $90 \times 45 \times 40$ was generated in the streamwise, vertical and spanwise directions, respectively. This selection is the result of a grid dependency study (He *et al.*, 1999), where grid sizes of $70 \times 45 \times 30$, $80 \times 45 \times 40$, and $90 \times 50 \times 40$ have been tested. The grid structure and coordinate system used in the present study are shown in Figure 4.

The main computational parameters in the GA are as follows:

$$n = 5, frac = 0.05, max\_generation = 150, m = 20$$

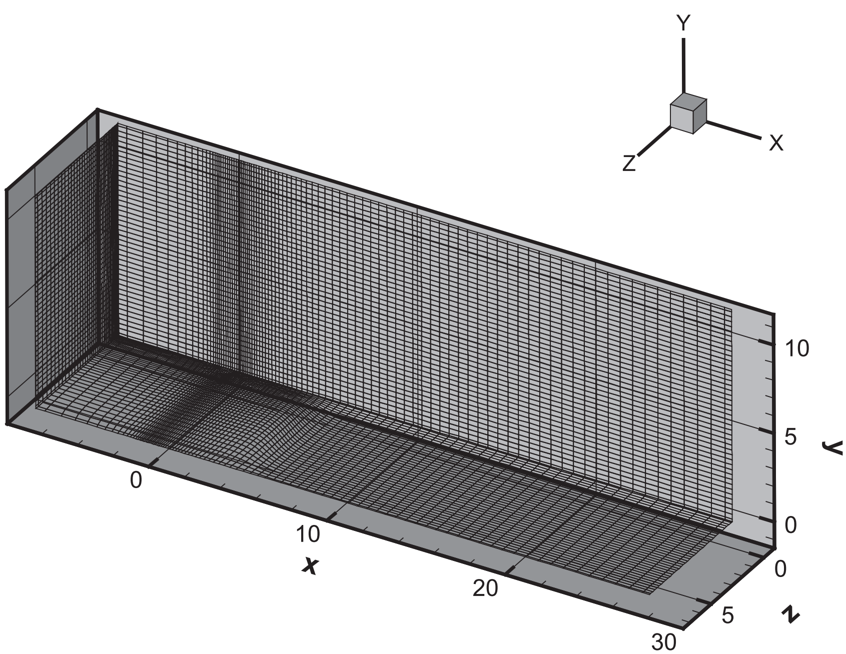
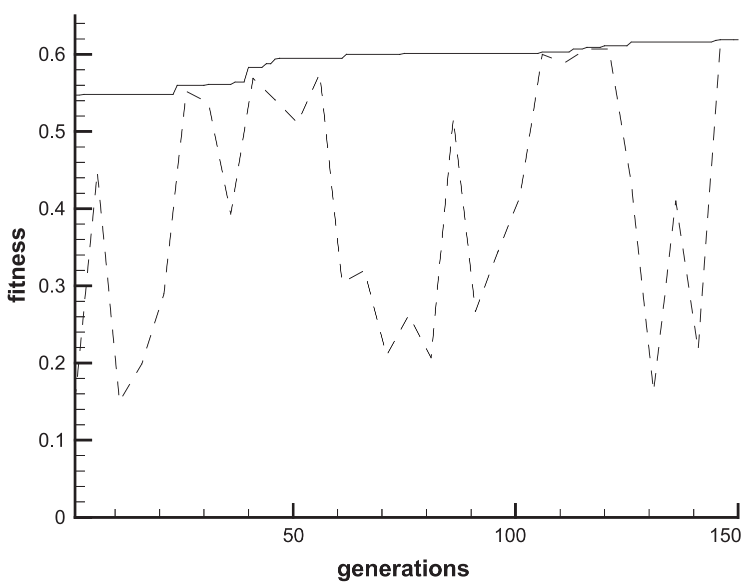


Figure 4.
Computational grid and coordinate system

where n denotes size of population, *frac* is the criteria used to detect population convergence, m denotes total number of positions at which the computation result is compared with experimental data, and *max_generation* denotes how many generations the population will evolve. Two bigger sizes of the population, $n = 10$ and $n = 20$, have been tested for the optimization problem. They gave almost the same results, so the parameter of $n = 5$ for this problem is recommended and the corresponding results are presented here. That means five fully converged simulation of mass transport in one generation. Total 150 generations were used. So total 750 fully converged simulation of mass transport are used in the calibration.

A steady flowfield was obtained first by using a constant Schmidt number of 0.8, which is commonly adopted in jet-in-crossflows computation. Because for the cases studied here the temperature difference between the jet and the crossflow is small and temperature variation has very little effect on the velocity field, only the enthalpy equation was solved during the optimization process, with other flow variables unchanged.

Figure 5 shows the convergence history of the genetic algorithm through generations, where the history of average individual fitness is shown every five generations. It shows that the best individual fitness keeps increasing through generations. The history of average individual fitness shows that there are several restarts of the micro-GA. It demonstrates that the micro-GA indeed prevents premature convergence. Figure 6 shows the history of three model parameters through generations. It can be seen that there are big jumps of these parameters through generations. Through 150 generations, the three model parameters are optimized as:



Note: Solid line = best individual fitness; dashed line = average individual fitness

Figure 5.
Convergence of the genetic algorithm through generations

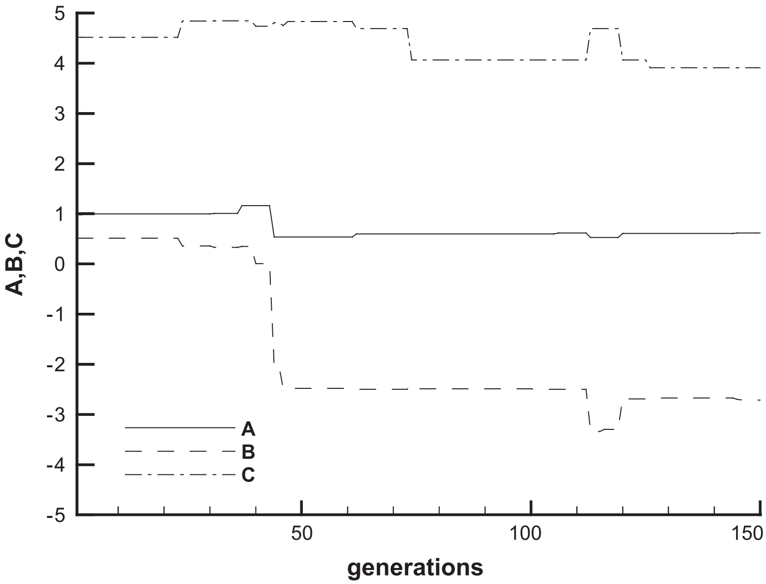


Figure 6.
Evolution of magnitudes of three model parameters through generations

$$A = 0.616, B = -2.713 \text{ and } C = 3.910.$$

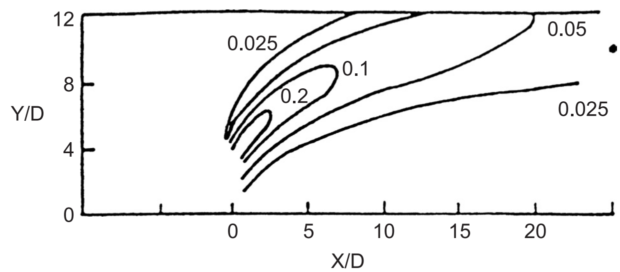
Figure 7 shows a qualitative comparison of temperature contours between the experimental data and numerical predictions, with optimized model parameters and with a constant Schmidt number of 0.8. These results show that predicted temperature profiles with proposed variable Schmidt number model agrees much better with experimental data than that with a constant Schmidt number of 0.8. Quantitatively, the fitness value using the optimized variable Schmidt number model is 0.62, while that using a constant Schmidt number of 0.8 is only 0.294.

Figure 8 shows the distribution of predicted Schmidt number at center-plane with optimized model parameters. It shows that the predicted Schmidt number is far from uniform and the magnitude of the Schmidt number is much lower than 0.8. This demonstrates that the variable Schmidt number model is necessary for calculating the mixing process in jet-in-crossflow.

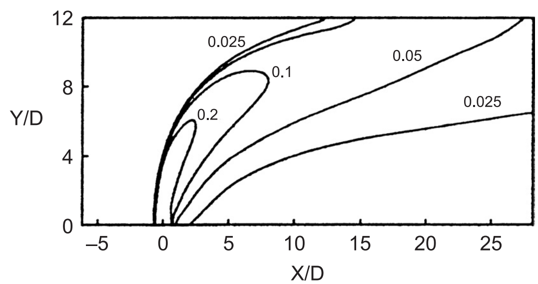
It should be noted that the model parameters were optimized to maximize the agreement of the predicted temperature profile with experimental profile. More work is needed to validate the calibrated parameters, and we proceed with the validation using three different sets of experimental data.

Validation

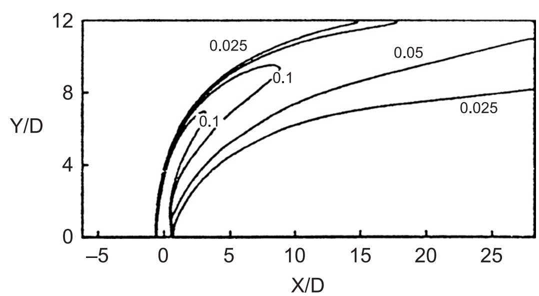
In the previous section, we have selected the model constants based on one set of experimental data through the application of Gas. In order to confirm that the model so determined is generally applicable to a class of jet-in-crossflow problems, we apply the model to the prediction of other jet-in-crossflows for which experimental data exist. Three sets of experimental data are selected for this validation.



(a) Experimental Data[a]



(b) Variable Schmidt number model



(c) Constant Schmidt number Sch=0.8

Figure 7. Non-dimensional temperature distributions in the symmetric plane

Source: [a]Kamotani and Greber (1974)

(1) *Kamotani and Greber (1972, 1974)*. First, we have used the other two momentum ratio, J = 8 and J = 72 of Kamotani and Greber's, as a validation of the new proposed variable Schmidt number model. The geometry and simulation are the same as J = 32's except the momentum ratio. The comparison of the temperature distribution at the central symmetric surface between experimental data and the numerical simulation with both constant Schmidt number 0.8 and the variable Schmidt number model was made. The results are shown in Figures 9 and 10 and show that the new proposed model gave a better prediction of the temperature distribution than constant Sc 0.8.

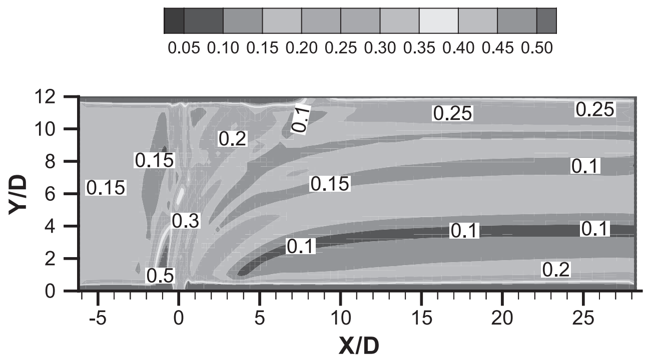


Figure 8.
Contours of predicted Schmidt number in the symmetric plane

(2) *Crabb* et al. (*1981*). To verify the observation further, we use another set of experimental data obtained by Crabb *et al.* (1981) and again use the variable Schmidt number model and the Sc = 0.8 to calculate the scalar field in a jet-in-crossflow. In this case, Crabb *et al.* (1981) used Helium trace to identify the species concentration distribution in this isothermal flow. Figure 11 presents the comparison between measured and calculated species concentrations using the variable Schmidt number model and the constant Schmidt number of 0.8 at four downstream locations of X/D = 4, 6, 8 and 10. It again can be seen that the predicted species concentration profiles using the proposed variable Schmidt number model better match the measured data.

(3) *Sherif and Pletcher (1989a, b)*. The experimental data of a single heated round jet issued into a confined crossflow by Sherif and Pletcher (1989a, b) was selected for the third validation. The distance from the jet exit to the opposite wall is H = 18D for velocity ratio R = 4. The velocity of the water crossflow is 0.4m/s.

The variable Schmidt number model is applied to this case using the same $90 \times 45 \times 40$ grid. In Figure 12, we have plotted vertical profiles of velocity magnitude on the center-plane at four streamwise locations. The data of Sherif and Pletcher (1989a) for the same locations are plotted as symbols on the same figures. Overall, the qualitative agreement between simulated and experimental results is very good. The simulation reproduces the two local maxima observed in the experiment in each downstream profile, and correctly predicts the evolution of the velocity profile. Quantitatively the comparison is also satisfactory. The prediction agrees with experimental measurements in the location of the maximum magnitude for each vertical profile, although the maximum velocity is slightly under-predicted. The discrepancies between numerical predictions and experimental data are believed to be caused by the deficiencies of the standard k-ε model.

In Figure 13 we present vertical mean scalar profiles on the center-plane at four streamwise locations. The data of Sherif and Pletcher

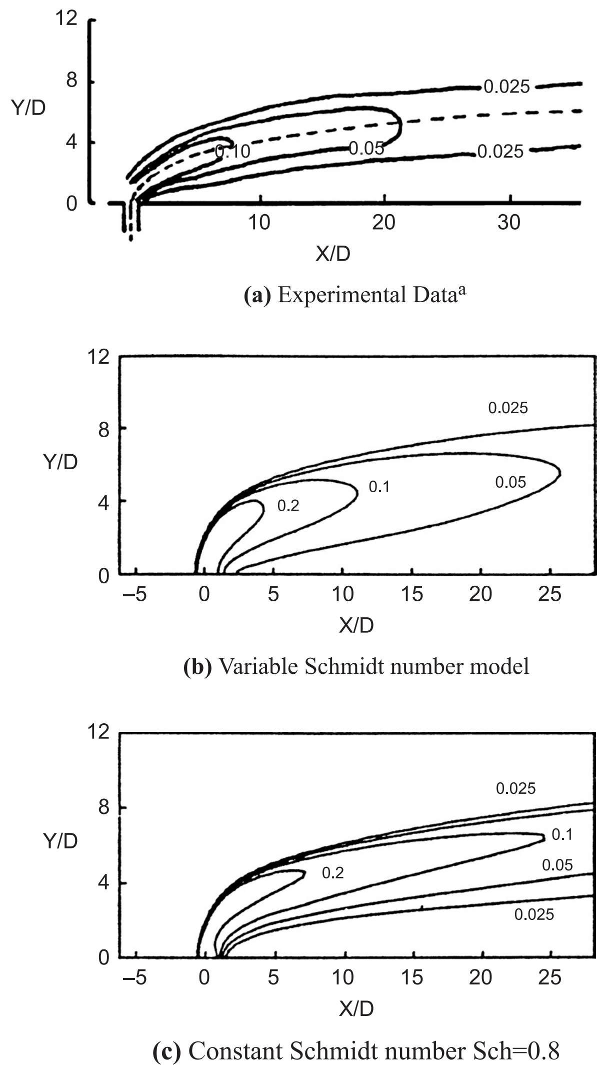


Figure 9.
Non-dimensional temperature distribution in the symmetric plane (J = 8)

Source: [a]Kamotani and Greber (1974)

(1989b) for the same locations are plotted as symbols on the same figures. Both the qualitative and the quantitative agreements between numerical prediction and experimental data are very good. The prediction agrees well with experimental data in the location and value of the maximum magnitude for each vertical profile.

These results demonstrate that the proposed variable Schmidt number model with the optimized model parameters is indeed appropriate for the simulation of turbulent scalar mixing in jet-in-crossflows.

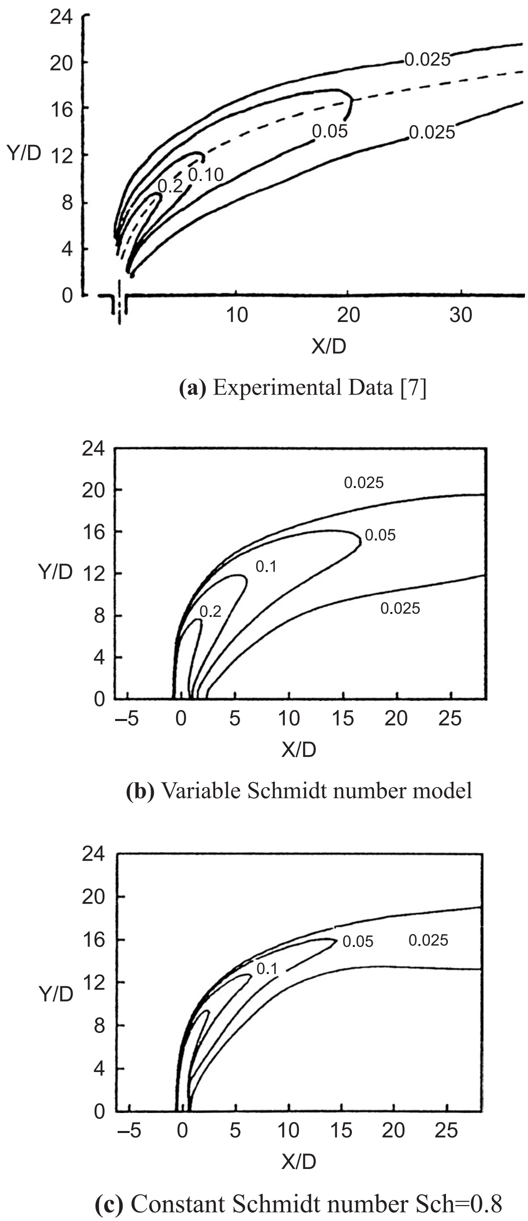
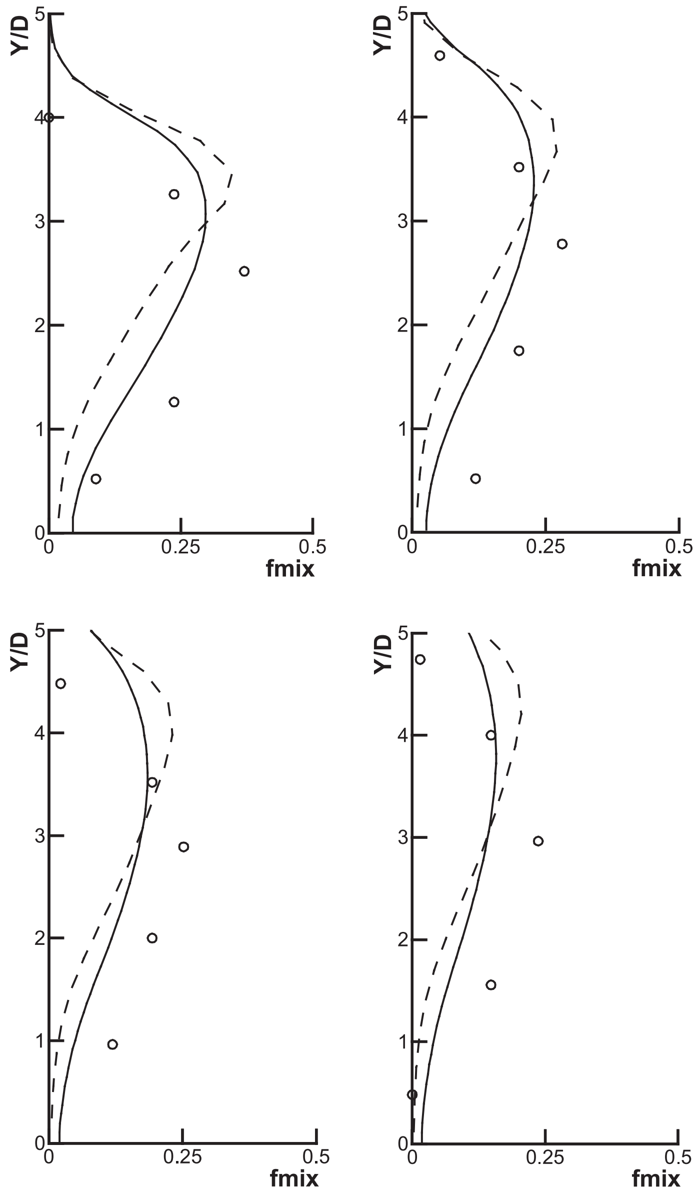


Figure 10.
Non-dimensional temperature distribution in the symmetric plane (J = 72)

Conclusions

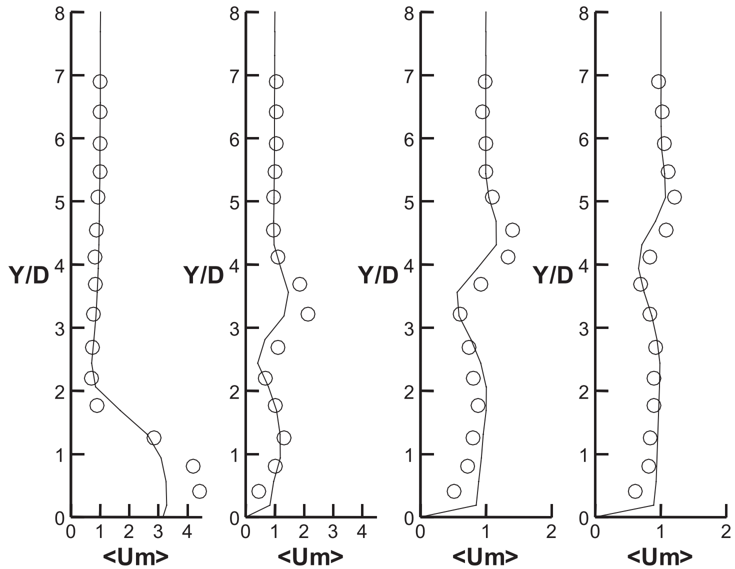
The transport equation of the turbulent scalar flux is analyzed and simplified under some assumptions. A variable Schmidt number model with three unknown model parameters is derived. Since the variable Schmidt number model is nonlinear and the large parameter space is multimodal, calculus-based and enumerative techniques were deemed inappropriate for either robustness



Note: Solid line: variable Schmidt number model;
Dashed line: Sc=0.8; Circle: Experimental data.

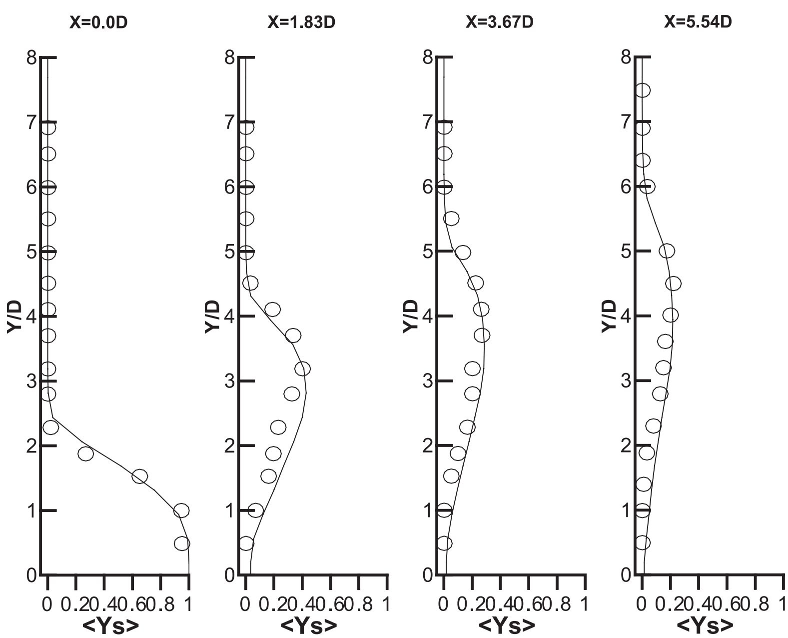
Figure 11.
Species concentration distribution at the jet center plane

or efficiency. A uniform micro-GA is implemented to determine the unknown parameters. This is the first known application of the GA technique to turbulence modeling. Overall, the GA technique worked exceptionally well for this problem in a cost-effective and time-efficient manner.



Note: Solid line, computation; open circles, Sherif and Pletcher (1989a)

Figure 12.
Vertical profiles of velocity magnitude in the symmetric plane



Note: Solid line, computation; open circles, Sherif and Pletcher (1989b)

Figure 13.
Vertical profiles of mean scalar concentration in the symmetric plane

A set of experimental data of a single round jet issued into a confined crossflow is selected for calibration of three model parameters. Three sets of experimental data of jet-in-crossflows are used for the model validation. Numerical results show that, compared to the use of a constant Schmidt number, the proposed variable Schmidt number model gives better predictions for scalar mixing in jet-in-crossflows. The present work demonstrated the applicability and usefulness of GA in the development of turbulence models in general.

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